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PROKON
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LONG TERM DEFLECTION USING PROKON

PROKON

ABSTRACT

Continuous Beam/Slab Design and Rectangular Slab Design are two modules in PROKON's concrete design suite that consider the effects of long-term deflection when designing reinforced concrete members. This article will discuss the calculation of long-term deflection based on the guidance of two international design codes, how the PROKON modules have been laid out in accordance with these codes and common user concerns to note when using the design modules.

INTRODUCTION

The prediction of long-term deflection is important in the design of concrete members for satisfactory performance during its use. Excessive deflection of a structure or part thereof adversely affects the appearance and efficiency of the structure, finishes or partitions. Long-term deflection in concrete members is defined as the deformation that occurs over time due to creep and drying shrinkage, and the deflection of transient loads after concrete cracking has occurred.

Various factors such as the age of loading, humidity and temperature ranges during curing, concrete mix proportions and the strength of aggregate in the concrete mix, all have an effect on the amount of creep induced in the structure.

Drying shrinkage is the contraction of a hardened concrete mixture due to the loss of capillary water. When concrete is restrained at its end conditions or by reinforcement, drying shrinkage causes an increase in tensile stress, which may lead to cracking and internal warping before the concrete is subjected to any kind of loading. The shrinkage deflection in structural concrete depends primarily on the amount and position of reinforcement in a member. It is the presence of non-symmetrical reinforcement within the depth of member that causes the member to warp towards the face with more reinforcement.

Various international design codes have derived relevant means to quantify the effect of long-term deflection on structural concrete. This article will discuss fundamental principles from these design codes which PROKON has adopted within the concrete design suite to estimate the effects of long-term deflection in concrete members.

DESIGN APPROACH

1. CRACKED SECTIONS

Design codes typically suggest using an elastic analysis to determine the moments and forces of members subjected to their loadings at serviceability limit states, and then assess the extent of concrete cracking. Where a single value of stiffness is used to characterize a member, the member's stiffness may be based on the cross section. In order to determine the long-term deflection, the curvature of the member is to be calculated according to the defined cracked or uncracked state of the member's cross section. When calculating the curvature, it is essential to determine the state of the concrete section taking note of the following assumptions:

Cracked section assumptions:

- Strains are calculated on the assumption that plane sections remain plane;
- The reinforcement in tension or compression is assumed elastic with a modulus of elasticity of 200 GPa;
- The concrete in compression is assumed elastic and under long-term loading an effective modulus may be taken having a value of $1/(1+\Phi)$ times the short-term modulus where Φ is the appropriate creep coefficient;
- To account for tension stiffening, the software follows the approach of BS 8110-1985 (and earlier editions and other derived codes). Stresses in the concrete in tension are to be calculated assuming the stress

distribution to be triangular, having a value of zero at the neutral axis and a value at the centroid of the tension steel of 0.55 MPa when considering the long-term effects and a value of 1.0 MPa when considering the short-term effects.

Uncracked section assumptions:

- The concrete and steel are considered fully elastic in tension and compression. The elastic modulus for both steel and concrete are derived in the same way as that of the cracked section.

Once the state of the section is determined, a new cracked depth and second moment of area is calculated under the assumption that when the section is cracked, the tensile stress in the concrete is equated equal to either 1MPa (in the short term) or 0.55 MPa (in the long term) and when the section is not cracked, the tensile stress in the concrete cannot exceed 1.0 MPa. Thereafter the relevant curvature formulae is used to determine the shrinkage, creep and instantaneous curvature at certain intervals along the members length. These three components of curvature are calculated as follows:

- The creep curvature is calculated using the total permanent load (Total dead load and the portion of the transient load which is considered as permanent);
- The instantaneous curvature is calculated by subtracting the permanent load’s instantaneous curvature from the total load’s instantaneous curvature;
- The shrinkage curvature is calculated considering the sections free shrinkage strain, modular ratio and first moment of area about the centroid of the cracked or gross section.

The creep, instantaneous and shrinkage curvature is there after added together to determine the total curvature at certain intervals along the members length. The total long-term curvature of the section is then numerically integrated to determine the total long-term deflection.

2. EQUIVALENT MOMENT OF INERTIA

Branson's formula (e.g. ACI318-14 c*lause 24.2.3.5)* and variations of it (e.g. EN 1992-1-1:2004 clause 7.4.3(3)) expresses the effective section stiffness of a cracked beam. The immediate deflection can be calculated by elastic analysis using an effective moment of inertia (I_e) not greater than (I_g).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 * I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) * I_{cr} \qquad \textit{(ACI 318-14 clause 24.2.3.5)}$$

Where

$$M_{cr} = \frac{f_r * I_g}{y_t}$$

- M_{cr} is the cracking moment;
- M_a is the maximum moment of the member at the stage the deflection is calculated;
- I_{cr} is the moment of inertia of a cracked section;
- f_r is the modulus of rupture (the tensile stress at which cracking occurs due to flexure);
- I_g is the moment of inertia of concrete gross section neglecting reinforcement;
- y_t is the distance from the centroidal axes of cross section to the extreme fiber in tension.

For the additional long-term deflection resulting from creep and shrinkage of flexural members, ACI 318-14 requires multiplying the immediate deflection due to sustained load by the factor

$$\lambda = \frac{\xi}{1+50*\rho'}$$ *(ACI 318-14 clause 24.2.4.1.1)*

Where

- $\rho' = \frac{As'}{bd}$ is the ratio of compression reinforcement at midspan for simple and continuous beams as well as at the supports of cantilevers;
- As' - the area of the compression reinforcement;
- b - the width of compression face of number;
- d - the distance from extreme compression fiber to the centroid of tension reinforcement;
- ξ (In PROKON ε is used) - a time dependent factor equal to 2.0, 1.4, 1.2, or 1.0, respectively for 5 years or more, 12, 6, and 3 months.

PROKON'S APPROACH TO LONG TERM DEFLECTION

1. CONTINOUS BEAM/SLAB DESIGN APPLICATION

PROKON's Continuous Beam/Slab Design module calculates the long-term deflection as follows:

1. It uses the results from the elastic analysis at serviceability limit states (SLS) to determine the state (cracked or uncracked) of the cross section at certain increments along the members length. The user is able to view the crack file (cb.def which is exported to the users working folder) to evaluate the state of the sections along the members length.
2. Once the state of the section is determined, a new cracked depth (or position of the neutral axis) and second moment of area is calculated under the assumption that when the section is cracked, the tensile stress in the concrete is equated equal to either 1MPa (in the short term) or 0.55 MPa (in the long term). It then uses this information to determine the required reinforcing to cater for the relevant bending moments and shear forces.
3. Thereafter it allows the user to generate a reinforcing layout using the required reinforcing computed by the program. The user is also able to adjust the layout, type and size of the reinforcing to meet the relevant design specifications and industry standards.
4. Using either the required or entered reinforcing (labelled as such when the user has altered the reinforcing layout), the program will then determine the long-term using results obtained from the analysis at serviceability limit states (SLS).

The long-term deflection is then computed using the numerical integration of the curvature (M/EI) diagrams and thereafter returns a graph indicating the shrinkage, creep and instantaneous components of the total long-term deflection.

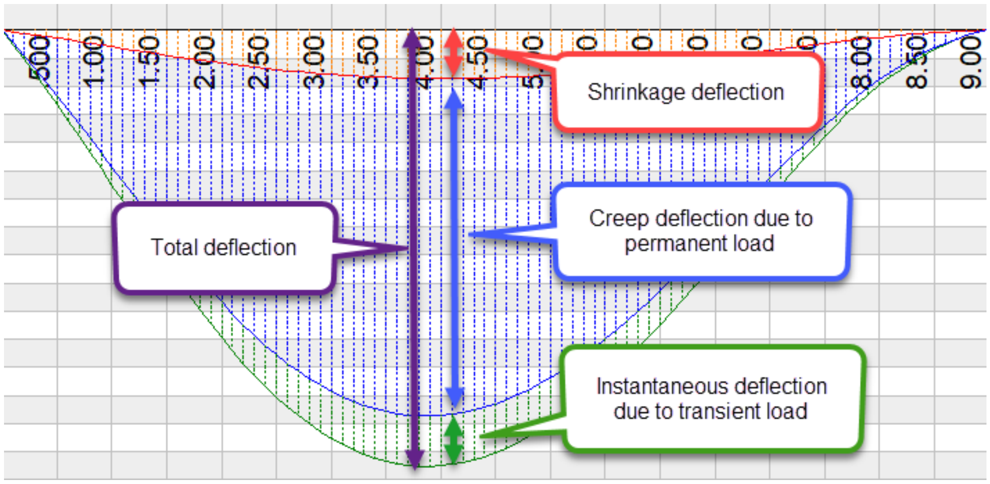

FIGURE 1: GRAPH INDICATING COMPONENTS OF THE TOTAL LONG-TERM DEFLECTION

2. RECTANGULAR SLAB PANEL DESIGN APPLICATION

An application of the ACI method is found in the PROKON Slab (Rectangular Slab Panel Design) module. The program calculates the elastic short-term deflection for each load case using a grid of finite element analysis. It uses the gross uncracked concrete section for the stiffness calculation. When calculating long-term deflections, the program adjusts the stiffness of each finite element in the slab analysis to reflect its level of cracking, creep and shrinkage using the approximate method outlined in ACI 318. The program follows this approach for the deflection calculation irrespective the code selected for the design.

TYPICAL EXAMPLES USING PROKON

The application of the British Standard's calculation is found in PROKON's Continuous Beam/Slab Design module. When defining the inputs, the program can calculate the percentage of live load considered as permanent and the program will do pattern loading automatically. Beams and one-way spanning slabs can be modelled in CB for analysis, design and detailing purposes.

It is possible to do the design of a flat slab in Continuous Beam/Slab Design as well. A quick way to get the inputs of the flat slab is to define sub-frame strips in PROKON Sumo. The geometry as well as applied loading can be exported to Continuous Beam/Slab Design for analysis, design and detailing.

Concrete beams that have been analyzed in PROKON analysis programs can be exported to Continuous Beam for detailing purposes. Note that this process uses the results that were calculated in the analysis program. Therefore, pattern loading must be applied to the loading in the Analysis software. The beam's long-term deflection can also be calculated within Continuous Beam using either the required or entered reinforcement.

When using Continuous Beam, the user is to take cognisance of the following when designing concrete members:

- Excessive creep is often due to insufficient section depth for continuous beams. This can be altered by using the suggested L/D ratios in the relevant design codes.
- Adding a considerable amount of bottom reinforcing will cause an increase in the amount of shrinkage deflection which in tern will have an effect on the overall long-term deflection.
- If no tension reinforcement is specified in the cracked zone, then the program reverts back to using the required reinforcement. The user is required to specify tensile reinforcement in order to evaluate the effect of the user defined entered reinforcing.
- Increasing the load will increase the long-term deflection disproportionally. This is due to the fact that the program uses the crack file to calculate the reinforcing demand at ULS. This in tern effects the cracking strength of the member at SLS.
- The program does not use the entered reinforcing for ribs (5), and column (3) and middle (4) strips to calculate LTD as it is incorrect to consider such elements in isolation when evaluation deflections. (Refer to clause 3.7.2 of BS 8110 – 2: 1985). The program does however give a rough estimate on the average LTD across the slab.

SUMMARY

When designing members such as concrete beams and slabs, it is important to consider what effect long-term deflection might have on the member being designed. Various international design standards have specified long - term deflection limits to which the Engineer is to consider when designing concrete members. PROKON'S Continuous Beam and Slab modules allow the user to evaluate the magnitude of long – term deflection through its interactive analysis, design and detailing processes.

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Prokon Software Consultants (Pty) Ltd

www.prokon.com

Andrew Brooks

Michael Klomp