Interpreting monolithic beam results

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Abstract–**Few problems in structural engineering attract as much attention as the modelling of flanged beams. The interaction between a beam and a slab cast monolithically is a complex 3D problem. A full 3D solution is possible with modern computer hardware – however, these have proven to be too complicated, while the processing of results is too laborious for routine work. For this reason, we try to simplify the situation by using a combination of 2D shell and 1D beam elements instead. The monolithic beam function in Sumo is at the heart of this discussion.**

I. Introduction

MONOLITHIC BEAMS are used in situations where a beam and a slab are cast in the same concrete pour and is intended to act as a unit. The slab is modelled with a series of 2D shell elements while the beam is modelled with 1D beam elements. The beam elements are shifted vertically from the slab and connected with rigid links. Because of the vertical offset, an axial force is introduced in the beam. To balance the force in the beam, the shell elements develop a membrane stress.

II. Modelling Options

Modelling a flanged beam in Sumo can be done in various ways, note the location of nodes for each option

Fig. 1. Typical T-beam to be modelled.

A. Beam elements

The T-beam is modelled as a series of beam elements.

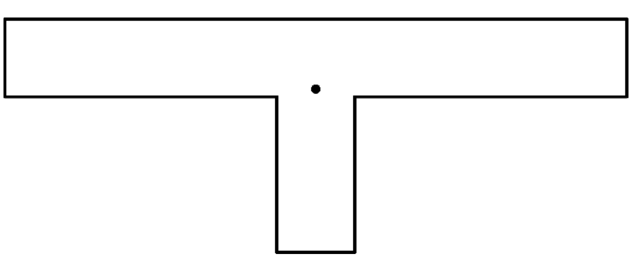

Fig. 2. T-beam modelled with beam elements.

B. Beam and shell elements—no offset

Using a slab and beam in combination. The slab, or flange, is modelled using several shell elements together with beam elements that represent the web of the T-beam.

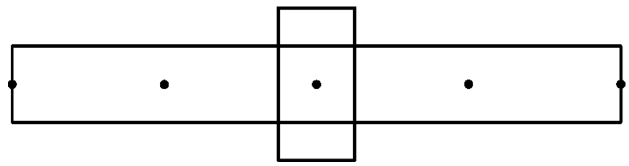

Fig. 3. T-beam modelled with a combination of beam and shell elements—no offset.

C. Beam and shell elements—with offset (Monolithic beam)

Using a slab and offsetting the beam using rigid links. This model can be done by the Monolithic beam function in Sumo.

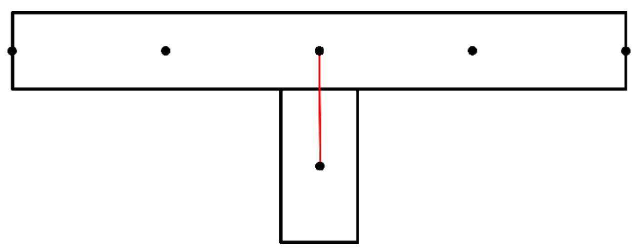

Fig. 4. T-beam modelled with a combination of beam and shell elements. The beam elements are moved vertically and connected with rigid links.

III. Results

A simply supported beam subjected to a UDL will yield different stress components depending on the model.

TABLE 1

Expected result components for each model.

Model	Beam elements			Shell elements		
	Shear	Moment	Axial	Out-of-plane		In-plane
				Shear	Moment	
A	X	X	-	-	-	-
B	X	X	-	X	X	-
C	X	X	X	X	X	X

IV. Dimensioning the Beam

To determine the required reinforcing for a beam, it is important to consider a section of the beam at various critical locations along the length. Typically, a concrete beam is designed for a maximum shear force and moment. It is required to sum the beam and shell results to arrive at a total shear and moment value which can be used to design a cross-section.

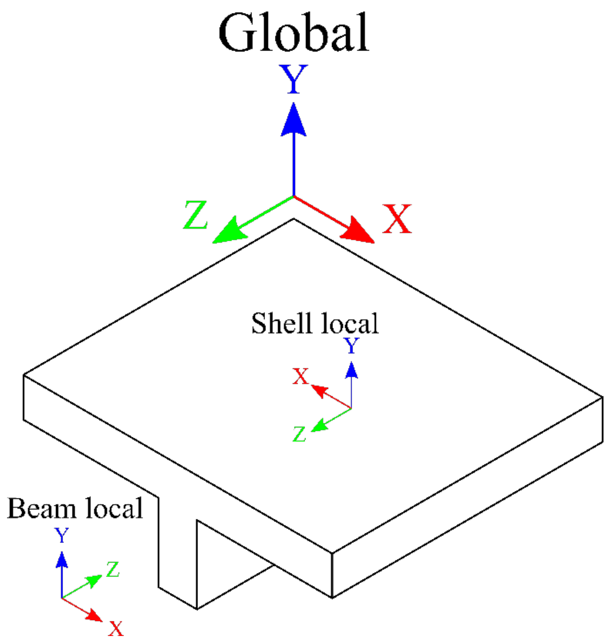

Fig. 5. Axis orientation used to construct the following equations.

Model A

$$V_{total} = V_{beam} \quad (1)$$

$$M_{total} = M_{beam} \quad (2)$$

where

- V_{beam} is the shear force in the beam.
- M_{beam} is the moment in the beam.

Model B

$$V_{total} = V_{beam} + \int_{flange} V_y \, dx \quad (3)$$

$$M_{total} = M_{beam} + \int_{flange} M_x \, dx \quad (4)$$

where

- V_y is the shear force at nodes between shell elements.
- M_x is the moment at nodes between shell elements.

Model C

$$V_{total} = V_{beam} + \int_{flange} V_y \, dx \quad (5)$$

$$M_{total} = M_{beam} + N_{beam} \, e_b + \int_{flange} (M_x + S_y \, e_p) \, dx \quad (6)$$

$$M_{total} = M_{beam} + N_{beam} \, e + \int_{flange} M_x \, dx \quad (7)$$

where

- N_{beam} is the axial force in the beam.
- e_b is the distance between the centroid of the web and the centroid of the T-beam.
- S_y is the in-plane stress of the shell elements.
- e_p is the distance between the centroid of the flange and the centroid of the T-beam.
- e is the distance between the centroid of the flange and the web.

Equation (6) takes the sum of moments about the centroid of the T-beam, and (7) about the centroid of the flange. Both will give the same results, although (7) is the easiest to use as it involves fewer components.

V. Example

To understand the different result components, consider the following example. A simply supported beam with a span of 10 metres loaded with a 10 kN/m UDL was analysed. Where shell elements were used for the flange, the load was distributed uniformly over the area. The dimensions of the flange and web are (b =) 2000 x (h =) 250 mm and (b =) 250 x (h =) 500 mm respectively. The results of the analysis are shown in Table 2. Detailed results from Model C can be found in Chapter VI.

TABLE 2
Summary of results from Sumo. Tabulated shear values are at supports while moment and deflection values are at midspan.

Member	Component	Unit	A	B	C
Beam	V	kN	50.00	29.45	39.98
	M	kN.m	125.00	61.60	17.51
	N	kN	-	-	241.38
Shell	V_{y1}	kN/m	-	7.50	3.40
	V_{y2}		-	9.69	-20.32E-4
	V_{y3}		-	9.02	12.02
	V_{y4}		-	9.69	-20.32E-4
	V_{y5}		-	7.50	3.40
	M_{x1}	kN.m/m	-	31.93	8.68
	M_{x2}		-	31.70	8.51
	M_{x3}		-	31.46	8.24
	M_{x4}		-	31.70	8.51
	M_{x5}		-	31.93	8.68
	S_{y1}	kPa	-	-	463.39
	S_{y2}		-	-	477.08
	S_{y3}		-	-	513.04
	S_{y4}		-	-	477.08
	S_{y5}		-	-	463.39
Total	V	kN	50.00	47.40	47.69
	M	kN.m	125.00	125.00	125.00
	Deflection	mm	-2.59	-9.86	-2.70

VI. Detailed Results of Model C

To calculate V_{total} or M_{total} anywhere along the length of the T-beam, the equations presented in Chapter IV can be used.

In this model, there are four shells along the width of the flange, all are 0.5 m square quadrilaterals. That means that *dx* will take on a value of 0.25 m at nodes 1 and 5 and 0.5 m at nodes 2, 3 and 4.

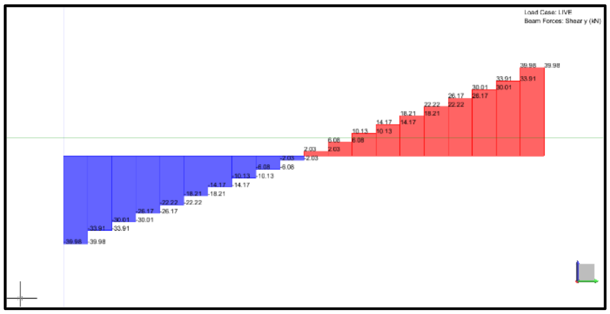

Fig. 6. Beam: Shear force in the beam (V_{beam})

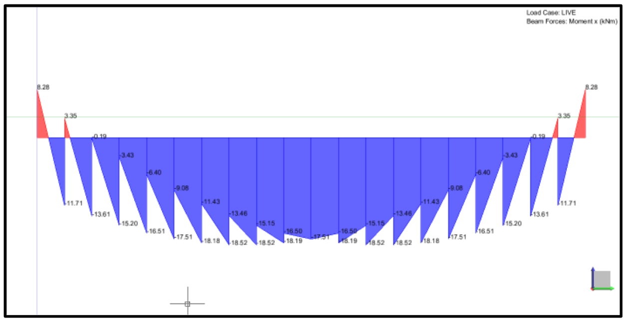

Fig. 7. Beam: Moment in the beam (M_{beam})

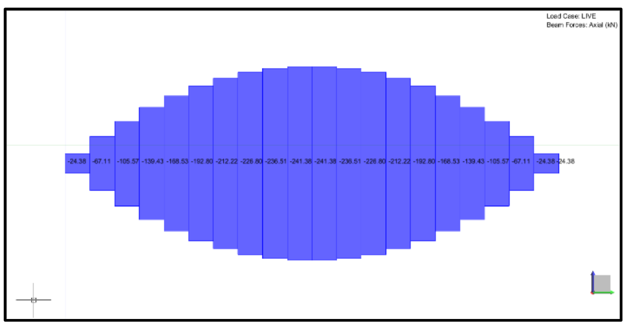

Fig. 8. Beam: Axial force in the beam (N_{beam})

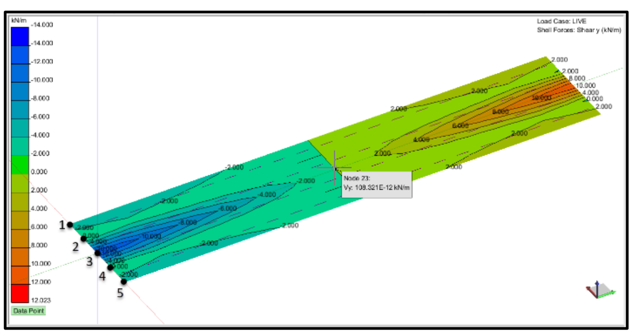

Fig. 9. Shells: Out-of-plane shear (Vy)

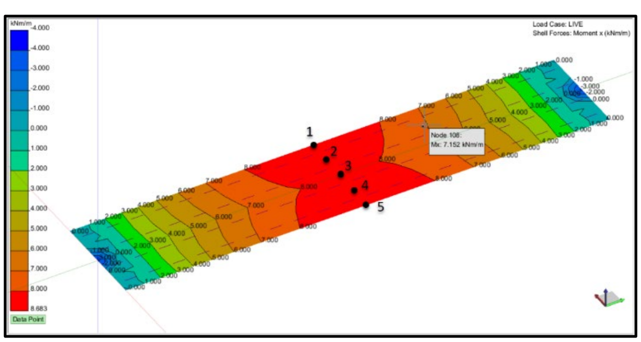

Fig. 10. Shells: Out-of-plane moment (Mx)

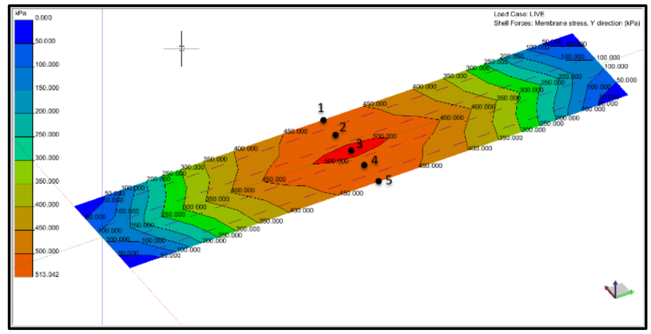

Fig. 11. Shells: In-plane stress (Sy)

Equation (5) can be used to determine V_{total}:

$$V_{total} = 39.98 + (3.4)(0.25) + (-20.32E\text{-}4)(0.5) + (12.02)(0.5) + (-20.32E\text{-}4)(0.5) + (3.4)(0.25) = 47.69\ kN$$

Equation (7) can be used to determine M_{total}:

$$M_{total} = 17.51 + (241.38)(0.375) + (8.68)(0.25) + (8.51)(0.5) + (8.24)(0.5) + (8.51)(0.5) + (8.68)(0.25) = 125.00\ kN.m$$

VII. CONCLUSION

The total shear force in Models B and C is not the same as in Model A. This is because the mesh size is not small enough. Reducing the mesh size will allow the shear to converge. The mesh was chosen deliberately large to simplify the presentation of results.

The example in this paper considers a sagging moment, i.e. tension in the web and compression in the flange. The same principles apply for a hogging moment, only the web will be in compression and the flange in tension.

The stepped shear force diagram (Fig. 6.) and the sawtooth bending moment diagram (Fig. 7.) are caused by the rigid links connecting the beam and shell elements. This is similar to the behaviour seen in the bottom chord of a Vierendeel truss.

Model A is computationally the cheapest way to model the beam. There are several limitations to this model, for example, shear lag in the flange cannot be seen. This option should be reserved for isolated beams or beams in a grillage.

Model B is not recommended because the stiffness of the beam is underestimated due to the web not being modelled at the eccentric position it will be in reality. Note that this model is prone to inaccuracy in the region where the shell and beam elements overlap because the area is considered twice. Accurate stiffness modelling is important, because stiffer elements attract more loading in a statically indeterminate structure. For instance, in a larger structure, Model A and C will show a greater moment than model B, because they are stiffer.

Model C is more complicated but produces good results. This model appears to be the best approximation of the increased stiffness of the T-beam as well as being adaptable to more complicated floor systems. Using the Monolithic beam function is the recommended way to model a flanged beam.